Simplified Solid Rocket Motor Design and Analysis

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Ultra Simplified Version:

Grain geometry and nozzle throat diameter determine burn area and Kn (burn area/throat area). Kn determines chamber pressure which determines the burn rate. Burn rate determines amount of propellant consumed during time increment under consideration and this establishes the burn area etc. for the next time increment.

Expanded Version:

$$Kn = \frac{A_b}{A_t} = \frac{gP_c}{r\rho C^*} = bP_c^{1-n}$$

Where:

A_b = Burn area (in^2)

A_t = Nozzle throat area (in^2)

g = gravitational constant 32.17 ft/s^2

P_c = Chamber pressure (psia)

r = Burn rate (in/s)

ρ = Propellant density (lb/in^3)

C^* = Characteristic Exhaust Velocity (ft/s)

b = Coefficient (Kn)

n = Burn rate exponent

Once burn area and throat area determine Kn, then Kn determines chamber pressure

$$P_c = cKn^{\frac{1}{1-n}} = \frac{r\rho KnC^*}{g}$$

Where:

c = Pressure coefficient

Chamber pressure determines burn rate

$$r = aP_c^n = \frac{gP_c}{Kn\rho C^*}$$

Where:

a = Burn rate coefficient

Most hobbyist grain/motor design programs utililize only the burn rate equation (determined by prior testing or, in some cases guessing), then combine r with density and C^*values determined using PROPEP or some other propellant evaluation program to determine Kn at two different pressures and use those values to determine the value of c in the pressure equation. They then use 1/(1-n) for the exponent value. In most cases experimental values of Kn vs. P_c do yield values for the pressure exponent very close to 1/(1-n) but not always. The experimental values for n depend on “accurate “ definitions of burn distance and burn time and these are not always as easy to define as it might appear at first glance.

To calculate thrust values for each pressure the motor experiences we have to use the expansion ratio of the nozzle and the specific heat ratio of the propellant to determine the vacuum thrust coefficient and after correcting this for ambient and chamber pressure, calculate the thrust for each pressure by

$$F = C_f P_c A_t$$

Where:

F = Thrust (lbs)

C_f = Thrust coefficient at that pressure